Universidad y Ciencia Trópico Húmedo
www.universidadyciencia.ujat.mx
28(3):271-277,2012

TWO NONLINEAR MATHEMATICAL MODELS TO ESTIMATE THE INTENSITY-DURATION-RETURN PERIOD OF RAINFALL EVENTS

Dos modelos matemáticos no lineales para estimar la intensidad-duración-período de retorno de eventos de lluvia

D Pereyra-Díaz

(DPD) Licenciatura en Ciencias Atmosféricas. Universidad Veracruzana. Circuito Gonzalo Aguirre Beltrán s/n. Zona Universitaria, Xalapa, Veracruz. CP 91090 México. dpereyra@uv.mx

Artículo recibido: 23 de enero de 2010, **aceptado:** 13 de octubre de 2012

ABSTRACT. In developing countries, many hydraulic engineering projects for the planning, designing, operation and protection against floods use intense rainfall data to estimate maximum runoff. In the case of Mexico, the only available rainfall data are usually those recorded with a pluviometer over 24 hours. The scarcity of data makes it necessary to validate and implement models to estimate short duration intense rainfall data. Two nonlinear mathematical models that estimate rainfall intensity based on the duration of the storm and the return period are proposed in this study. The models were adjusted to short duration rainfall data for the city of Xalapa (Veracruz), Cañon Fernández (Durango) and Cazadero (Zacatecas), and were compared with other models reported in the literature. The results show that: a) for model I, the determination coefficient (R^2) varied from 0.927 to 0.932 and the standard error of estimation (R_e) from 6.7 to 9.4 mm h^{-1}, and b) for model II, the R^2 varied from 0.934 to 0.988 and the R_e from 3.5 to 6.5 mm h^{-1}. The fit shows that model II is better than model I. In comparison, complex models like those of Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) were adjusted to extreme rainfall data for the city of Xalapa. The range of statistical trustworthiness of the models proposed in this study is of the same order of magnitude as that obtained by Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) and better than that obtained by other models (see Tables 1 and 2). The models proposed in this study must be calibrated before being applied to other regions, using a PC with statistical/mathematical software that may solve nonlinear equations.
Key words: Return period, *idf* relationships, rainfall intensity, short duration, nonlinear model.

RESUMEN. En países en desarrollo, muchos proyectos de ingeniería hidráulica con fines de planeación, diseño, operación y protección, contra inundaciones, usan datos de lluvias intensas para estimar los escurrimientos máximos. Para México en la mayoría de los casos, los únicos datos disponibles de lluvia son los acumulados en 24 h, medidos utilizando un pluviómetro. Debido a la escasez de registros se hace necesaria la validación, adecuación o la implementación de modelos para obtener las lluvias intensas de corta duración. En este estudio se proponen dos modelos matemáticos no-lineales, que estiman la intensidad de la lluvia con base a la duración de la tormenta y al período de retorno. Estos modelos fueron ajustados a datos de lluvia de corta duración, de la ciudad de Xalapa, Veracruz; Cañón Fernández, Durango y Cazadero, Zacatecas; y comparados con otros modelos existentes en la literatura. Los resultados muestran lo siguiente: a) Para el modelo I el coeficiente de determinación (R^2) varió entre 0.927 y 0.932 y el error estándar de estimación (R_e) entre 6.7 y 9.4 mm h^{-1}, b) Para el modelo II R^2 varió entre 0.934 y 0.988 y R_e entre 3.5 y 6.5 mm h^{-1}. El resultado del ajuste muestra, que el modelo II es mejor que el modelo I. Como comparación, modelos complejos como el de Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) fueron ajustados a lluvias severas de la ciudad de Xalapa. El rango de confiabilidad estadística de los modelos propuestos en este estudio es del mismo orden de magnitud que el obtenido para los modelos de Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) y mejor que otros modelos (ver Tabla 1 y 2). Los modelos propuestos en este estudio deben ser previamente calibrados para ser usados en otra región, siendo necesario contar con una PC con software estadístico/matemático que resuelva ecuaciones no lineales.
Palabras clave: Período de retorno, relaciones *idf*, intensidad de la lluvia, corta duración, modelo no lineal.

INTRODUCTION

Rainfall intensity in a storm is defined as the amount of water that falls over a given point of the surface per unit time; it is inversely proportional to the duration of the storm. The *duration* of the storm is the time elapsed from the moment it starts to rain to the time it ceases (Pereyra *et al*. 2004). Intensity-duration-frequency (*idf*) curves were defined by Willems (2000) as the existing relationship between mean rainfall intensity and the frequency of occurrence (i.e., inverse of the return period).

The *idf* relationship is widely used to protect the facilities, services and installations that are needed for the functioning of a community or society. According to Smith (1993) the analysis of the frequency is widely used to design hydraulic infrastructure (e.g. municipal sewer dumping systems, spillway, sewage, bridges and drainage systems for agricultural drainage) intended to control the runoff generated by storms. In addition, Willems (2000) stated that the *idf* of extreme rainfall events is used to estimate extreme floods when designing reservoirs. Bell (1969) and Koutsoyiannis *et al*. (1998) considered that the *idf* relationship is one of the most commonly used tools in engineering projects for the planning, designing, operation and protection against floods. The *idf* relationships was established in 1932 (Bernard, 1932). Many other relationships have been constructed in several parts of the world since then. The geographical distribution of *idf* relationships has been studied in several developed countries, and maps have been constructed to provide rainfall intensities or depths for various return periods and durations since the early 1960s (Koutsoyiannis *et al*. 1998). However, maps of rainfall intensity have not been constructed until recently in many other countries; in some cases one has to retrieve the original intensity records from a nearby rainfall-recording station to construct an *idf* relationship.

According to Koutsoyiannis *et al*. (1998) *idf* maps have been generated in the United States of America (USA), the United Kingdom, Italy, Australia, India and Namibia. In the USA, *idf* maps were first developed by the U.S. Weather Bureau in 1961, and later by the National Oceanic and Atmospheric Administration (NOAA). These maps have been reproduced in some manuals and textbooks of Hydrology including those of Chow (1964), Linsley *et al*. (1988), Raudkivi (1979), Viessman *et al*. (1989), Ponce (1989), Maidment (1993), Linsley (1994) and Chow *et al*. (1994). Maia & Fugagnolli (2001) published a validation and adjustment of the generalized intense rainfall equations that were proposed for Brazil by Bell (1969) and Chen (1983). After applying the equations to rainfall events of several cities of the state of Sao Pablo, Brazil, they stated that these equations could be used for localities with no rainfall records.

One of the few studies published on this subject in Mexico was carried out by Campos (1990); who obtained *idf* curves for Cazadero, Zacatecas, applying the equations of Bell (1969) and Chen (1983) of generalized intense rainfall. Pereyra *et al*. (2004) reviewed the existing models in the literature and fitted eight nonlinear models to the rainfall data of Xalapa, the capital city of the state of Veracruz. They obtained the best fit to the observed rainfall with the model of Koutsoyiannis *et al*. (1998), followed by those Chow *et al*. (1994) and Sherman (1931). Pereyra *et al*. (2005) also fitted Xalapa's maximum rainfall over 24 h to the equations of Bell (1969) and Chen (1983) in order to obtain maximum short duration rainfall. Two nonlinear mathematical models that estimate the intensity-duration-return period rainfall curves (idt) are presented in this study. These models are adjusted to intense precipitation for Xalapa, Veracruz (Pereyra *et al*. 2004), Cañon Fernandez, Durango, and Cazadero, Zacatecas (Campos, 1990), in Mexico.

MATERIALS AND METHODS

Proposed nonlinear mathematical models

Models I and II in this paper are proposed by analogy to existing models in the literature (Sherman, 1931; Bernard; 1932; Wenzel, 1982; Chow *et al*. 1994) (see Table 2), considering a) that storm intensity and duration are inversely related and b) that storm intensity and return period are directly related (Ponce, 1989). The novelty in these models is the introduction of the exponential term $exp(\eta d)$ in lieu of the term d^{η} used in other models (Table 2).

Model I. This model simultaneously relates intensity, duration and return period of rainfall in a family of idt curves (equation 1):

$$i = \frac{\lambda T^{\psi}}{exp(\eta d)} \quad (1)$$

where i is rainfall intensity in mm h^{-1}, d is the duration of the storm in minutes, T is the return period in years, and λ, ψ, η are nonlinear regression coefficients that vary with the place and the return period.

Model II. This model is a modified version of model I, where an additional parameter is added with the purpose of improving the estimation of rainfall intensity. As before, intensity, duration and return period of rainfall are related in a family of curves (equation 2):

$$i = \frac{\lambda T^{\psi}}{exp(\eta d) + \theta} \quad (2)$$

where θ is the new parameter that varies with the place and the return period; this parameter is estimated by successive approaches.

Method to fit the idt equations

The maximum rainfall data recorded for three localities in Mexico were used in order to fit equations (1) and (2) with observed rainfall data (e.g. Figures 1 and 2). As mentioned earlier, these localities were a) Xalapa, Veracruz for the 1927-2002 period; rainfall data were obtained for the most intense storm of each year (some years had missing data), b) Cañon Fernandez, Durango for the 1941-1978 period; rainfall data for this area were the most intense of each year, c) Cazadero, Zacatecas for the 1963-1978 period; rainfall data for this area were also the most intense of each year. Data for b) and c) waere taken from Campos (1990).

The first step in fitting the models was to transform the recorded maximum rainfall (mm) to intensity data (mm h^{-1}) for the durations of 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110 and 120 min. The second step was to place the maximum intensity of each duration time from highest to lowest in order to assign them their corresponding return period, T.

The models were fitted with the i, d and T values shown in Tables A.1 to A.3 (Pereyra *et al*. 2004). As the equations that describe each model are nonlinear and hyperbolic, the Quasi-Newton method of successive approaches (Nocodal & Wright, 2000; Fletcher, 2000) was used to solve them; this method is found in the nonlinear estimation module of the STATISTICA v.7 software (Statsoft 2004).

After adjustment, a residual analysis was carried out (R_e = observed intensity - intensity estimated by the model) in order to verify that the remainders had a random distribution (around the zero). Finally, the standard error of the estimation was calculated as the square root of the residual variance (Haber & Runyon, 1986).

RESULTS

Table 1 presents the parameters of models I and II for the adjustment to 1) extreme rainfall in Xalapa during the 1927-2002 period, 2) Cañon Fernandez, Durango during the 1941-1978 period, and 3) Cazadero, Zacatecas during the 1963-1978 period, together with the coefficient of determination R^2 (column 7) and the standard error of estimation R_e (column 8). Figures 1 and 2, and Table 1 make it possible to see that model II is adjusted best to the observed rainfall data in the three localities. Specifically, the best adjustment was that for Cazadero, Zacatecas, with an $R^2 = 0.988$; that is, the model explained 98.8 % of the cases (Figure 1). As a matter of fact, model I was also the best for Cazadero, Zacatecas, with an $R^2 = 0.932$, with the model explaining 93.2 % of the cases (Figure 2). In general, both models explained more than 91 % of the cases for the three localities.

Finally, Figures 3 and 4 present, as an example, the idt curves for the return periods of 2, 5, 10, 25 and 50 years and durations of 5 at 120 min, from models I and II adjusted to the observed rainfall of Cazadero, Zacatecas during the 1963-1978 period.

Table 1. Parameters of models I and II. Parameters are obtained from the adjustment to the observed intense rainfalls in Xalapa (Veracruz), Cañon Fernandez (Durango), and Cazadero (Zacatecas).
Tabla 1. Parámetros de los modelos I y II. Los parámetros fueron obtenidos del ajuste a las intensidades de lluvia observadas en Xalapa (Veracruz), Cañon Fernández (Durango) y Cazadero (Zacatecas).

Model	Locality	λ	θ	ψ	η	R^2	SEE, $R_e(mmh^{-1})$
$i = \frac{\lambda T^{\psi}}{exp(\eta d)}$	Xalapa, Veracruz	69.978		0.273	0.6170	0.927	6.7
	Cañón Fdez, Durango	79.360		0.237	0.0253	0.915	9.4
	Cazadero, Zacatecas	69.587		0.251	0.0160	0.932	8.8
$i = \frac{\lambda T^{\psi}}{exp(\eta d)+\theta}$	Xalapa, Veracruz	12.7030	0.835	0.274	0.1690	0.934	6.5
	Cañón Fdez, Durango	0.3463	-0.997	0.238	0.0156	0.981	4.3
	Cazadero, Zacatecas	0.4820	-0.996	0.210	0.0180	0.988	3.5

SEE = Standard error of estimation.

Table 2. Parameters obtained from existing models in the literature. Results correspond to the adjustment to extreme rainfalls of Xalapa City during the 1927-2002 period. The coefficient of determination R^2 and the standard error of estimation R_e (Taken from Pereyra *et al.* 2004) are also displayed.
Tabla 2. Parámetros obtenidos para modelos existents en la literature. Los resultados corresponden al ajuste de lluvias extremas de la ciudad de Xalapa durante el período 1927-2002. También se muestra el coeficiente de determinación R^2 y el error estándar de estimación R_e (tomado de Pereyra *et al.* 2004).

Authors	Model	λ	θ	ψ	η	R^2	SEE, $R_e(mmh^{-1})$
Ponce (1989)*	$i = \frac{\lambda}{d^{\eta}}$	269.876			0.446	0.473	18.7
Ponce (1989)*	$i = \frac{\lambda}{d+\theta}$	5362.611	50.607			0.491	17.8
Ponce (1989)*	$i = \frac{\lambda}{(d+\theta)^{\eta}}$	203548.7	108.451		1.625	0.493	17.8
Bernard (1932)	$i = \frac{\lambda T^{\psi}}{d^{\eta}}$	174.603		0.275	0.394	0.904	7.8
Sherman (1931)	$i = \frac{\lambda T^{\psi}}{(d+\theta)^{\eta}}$	120.794	1.388	0.274	1.378	0.934	6.5
Wenzel (1982)	$i = \frac{\lambda}{d^{\eta}+\theta}$	98.655	1.014		1.175	0.493	17.8
Chow *et al.* (1994)	$i = \frac{\lambda T^{\psi}}{d^{\eta}+\theta}$	70.218	0.935	0.274	1.12	0.934	6.5
Koutsoyiannis *et al.* (1998)	$i = \lambda \left\{ \frac{\psi - \ln\left[-\ln\left(1-\frac{1}{T}\right)\right]}{(d+\theta)^{\eta}} \right\}$	12046.41	82.71	3.036	1.368	0.988	2.7

SEE = Standard error of estimation. * Model referenced in Ponce (1989).

DISCUSSION

The adjustment of the models indicates that model II is better than model I. This is because the coefficient of determination for model II (R^2 = 0.988) was higher than that for model I (R^2 = 0.932), and its standard error of estimation (R_e = 3.5 mm h^{-1}) was lower than that obtained for model I (R_e = 6.7 mm h^{-1}). In comparison, the models of Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) were adjusted to extreme rainfall of Xalapa; their parameters and the standard error of estimation (R_e) are shown in Table 2. The range of statistical trustworthiness of the proposed models is of the same order of magnitude as that

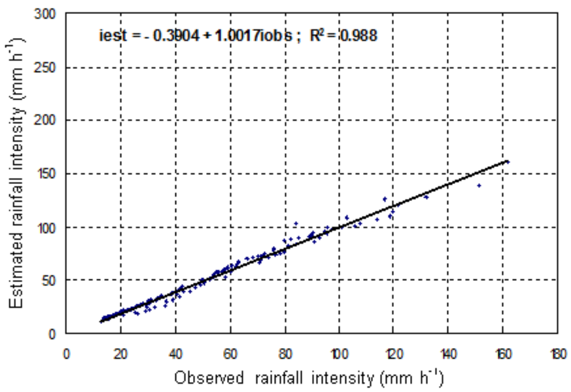

Figure 1. Correlation between observed and estimated rainfall intensities from model II for Cazadero (Zacatecas) during the 1963-1978 period.
Figura 1. Correlación entre intensidades de lluvia observada y estimada por el modelo II para Cazadero (Zacatecas) durante el período 1963-1978.

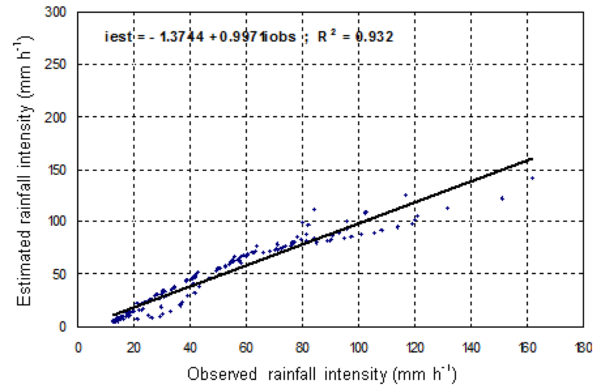

Figure 2. Correlation between observed and estimated rainfall intensities from model I for Cazadero, Zacatecas during the 1963-1978 period.
Figura 2. Correlación entre intensidades de lluvia observada y estimada por el modelo I para Cazadero (Zacatecas) durante el período 1963-1978.

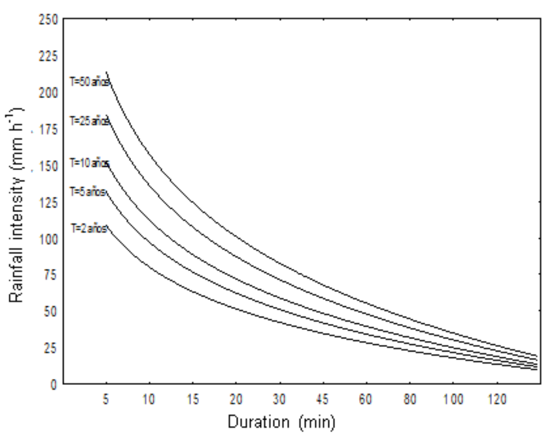

Figure 3. Intenisty-duration-period of return (idt) curves generated from model II for Cazadero, Zacatecas.
Figura 3. Curvas intensidad-duración-período de retorno (idt) generadas por el modelo II para Cazadero (Zacatecas).

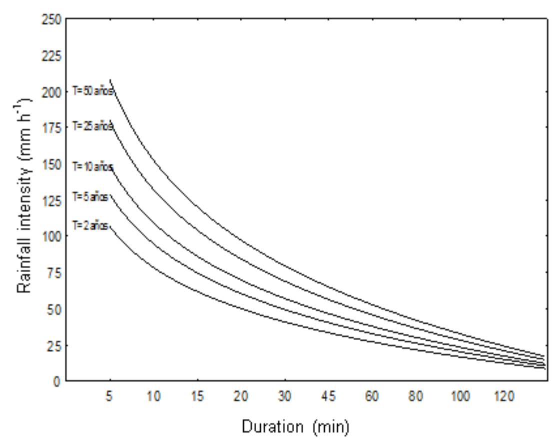

Figure 4. Intenisty-duration-period of return (idt) curves generated from model I for Cazadero, Zacatecas.
Figura 4. Curvas intensidad-duración-período de retorno (idt) generadas por el modelo I para Cazadero (Zacatecas).

obtained for Xalapa using the models of Koutsoyiannis *et al.* (1998), Chow *et al.* (1994) and Sherman (1931) (see Table 2) and better than that of other models. The models proposed in this study may be used if one has a PC with statistical/mathematical software that may solve nonlinear equations.

It is important to clarify that model I overestimates the intensities for small durations of 10 and 20 minutes. Models I and II fitted in this study may be cautiously used in the field of urban design for localities that lack continuous rainfall records over several years, but have climatic characteristics similar to those of the localities considered in this study.

ACKNOWLEDGEMENTS

The author expresses his gratitude to the National Meteorological Service of Mexico for sharing its rainfall records of Xalapa, Veracruz, and to Dr. Francisco Campos Aranda for making available the rainfall data of Cañon Fernandez, Durango and Cazadero, Zacatecas. Also expresses his gratitude to PhD. Alfredo Ruiz Barradas professor of Maryland University, for comments on the text.

CITED LITERATURE

Bell FC (1969) Generalizad Rainfall-Duration-Frecuency Relationships. Journal of Hydraulics Division ASCE 95: 311-327.

Bernard MM (1932) Formulas for rainfall intensities of long durations. Transactions Paper 1801 ASCE 96: 592-624.

Campos DF (1990) Procedimiento para obtener curvas I-D-Tr a partir de registros pluviométricos. Ingeniería Hidráulica en México 5 (2): 39-52.

Chen Ch (1983) Rainfall Intensity-Duration-Frecuency Formulas. Jounal of Hydraulic Enginneering 109(12): 1603-1621.

Chow VT (1964) Handbook of Applied Hydrology. McGraw-Hill Book Co., Inc., New York, USA. 1467 pp.

Chow VT, Maidment DR, Mays LW (1994) Hidrología Aplicada. Mc Graw-Hill. Interamericana, S.A Colombia, 584 pp.

Fletcher R (2000) Practical Methods of Optimization. John Wiley y Sons LTD, Great Britain, 436 pp.

Haber A, Runyon RP (1986) Estadística General. Addison-Wesley Iberoamericana, S. A., USA, 371 pp.

Koutsoyiannis D, Kozonis D, Manetas A (1998) A Mathematical Framework for Studying Rainfall Intensity-Duration-Frequency Relationships. Journal of Hydrology 206: 118-135.

Linsley RK, Kohler, MA Paulhus JL (1988) Hidrología para ingenieros, McGraw-Hill/Interamericana de México, 386 pp.

Linsley RK, JR (1994) Hidrología para ingenieros, McGraw-Hill, Inc., México, 386 pp.

Maia-Genovez A, Fugagnolli Pegoraro RC (2001) Análisis y evaluación de las ecuaciones de lluvias intensas generalizadas. Ingeniería Hidráulica en México XVI(3): 15-23.

Maidment DR (1993) Handbook of Hydrology, capítulo III, Precipitation, McGraw-Hill, Inc. primera edición, USA, pp. 1-47.

Nocodal J, Wright J (2000) Springer Series in Operations Research: Numerical Optimization. Springer-Verlag New York, Inc, 636 pp.

Pereyra-Díaz D, Pérez-Sesma JAA, Gómez-Romero L (2004) Ecuaciones que estiman las curvas intensidad-duración-período de retorno de la lluvia. GEOS 24(1): 46-56.

Pereyra-Díaz D, Ochoa-Martínez CA, Pérez-Sesma JAA (2005) Ecuaciones de lluvia intensa generalizada para obtener precipitaciones máximas de corta duración. GEOS, Bulletin of Mexican Geophisical Union 25(2): 340-351.

Ponce VM (1989) Engineering Hydrology. Principles and Practices, Prentice Hall, primera edición. New Jersey, USA, 640 pp.

Raudkivi AJ (1979) Hydrology. An Advanced Introduction to Hydrological Processes and Modeling, Pergamon Press Ltd., Headington Hill Hall, England, 479 pp.

Sherman CW (1931) Frequency and intensity of excessive rainfall at Boston. Mass. Transaction Paper ASCE 95: 951-960.

Smith JA (1993) Precipitation, In Handbook of Hydrology, Chapter III, McGraw-Hill, Inc. first edition, USA, pp. 1-47.

Statsoft (2004) STATISTICA (data analysis softwere system) version 7, Tulsa, OK. USA.

Viessman JR W, Lewis GL, Knapp JW (1989) Introduction to Hydrology, Harper y Row, Tercera edición. 704 pp.

Wenzel HG (1982) Rainfall for urban stormwater design, in Urban Storm Water Hydrology. Water Resources Monograph 7, American Geophysical Union. Washington, D.C., USA.

Willems P (2000) Compound intensity/duration/frequency-relationships of extreme precipitation for two season and two storm types. Journal of Hydrology, 233, 189-205.